

國立臺灣科技大學

八十九學年度博士班招生考試試題

系所組別：機械工程系甲一組、機械工程系甲二組、機械工程系乙組、機械工程系丙組、機械工程系丁組、
機械工程系戊組

科目：工程數學

1. Of all rectangular parallelepipeds which have sides parallel to the coordinate planes, and which are inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

determine the dimensions of that one which has the largest possible volume. (20%)

2. Reduce the quadratic form $A = x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$ to a canonical form by making an appropriate change in variables, $X = QX'$, where Q is an orthogonal matrix. (20%)

3. Find the complete solution of the equation

$$(1-x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 6(1-x^2) \quad (20\%)$$

4. The temperature T is maintained at 0° degree along three edges of a square plate of length 100 cm and the fourth edge (the top one) is maintained at 100° degree until steady-state conditions prevail.
- (a) Find an expression for the temperature T at any point (x, y) in the plate. (10%)
- (b) calculate the approximate value of the temperature at the center of the plate. (10%)

5. Find the integral $\int [(1+y)zdx + (1+z)xdy + (1+x)ydz]$ where c is

- (a) the triangle with vertices at $P_1(1,0,0), P_2(0,1,0), P_3(0,0,1)$ oriented from P_1 to P_2 . (10%)
- (b) a closed curve in the plane $x-2y+z=1$. (10%)

5

