

國立臺灣科技大學
八十九學年度博士班招生考試試題

系所組別：機械工程系甲一組
科目：彈性力學

- Given the following state of stress ($\sigma_{ij} = \sigma_{ji}$)

$$\sigma_{11} = -2x_1^2, \quad \sigma_{12} = -7 + 4x_1x_2 + x_3, \quad \sigma_{13} = 1 + x_1 - 3x_2,$$

$$\sigma_{22} = 3x_1^2 - 2x_2^2 - 5x_3, \quad \sigma_{23} = 0, \quad \sigma_{33} = -5 + x_1 + 3x_2 + 3x_3$$

(1) Determine the stress vector at point (x_1, x_2, x_3) on the plane $x_1 + x_2 + x_3 = \text{constant}$. (13%)

(2) What are the normal and shearing components of the stress vector at point $(1, 1, 3)$? (12%)
- (1) Explain why a given stress or strain field must satisfy the compatibility equation. (10%)

(2) The mathematical strains of an isotropic cantilever beam are given by

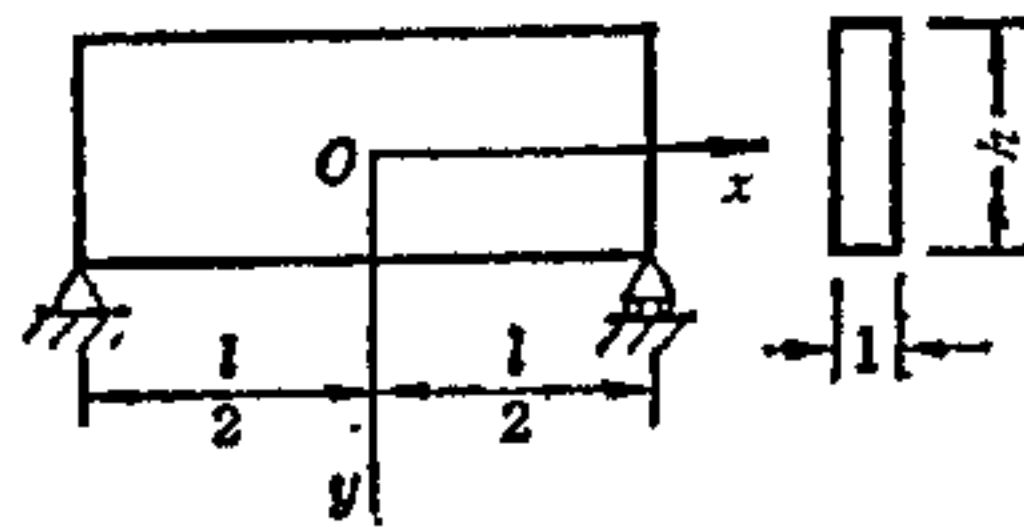
$$\epsilon_{11} = -\frac{Px_1x_2}{EI}, \quad \epsilon_{22} = \nu\frac{Px_1x_2}{EI}, \quad \epsilon_{12} = -\frac{(1+\nu)P}{2EI}(h^2 - x_2^2)$$

$$\epsilon_{22} = \nu\frac{Px_1x_2}{EI}, \quad \epsilon_{13} = \epsilon_{23} = 0$$

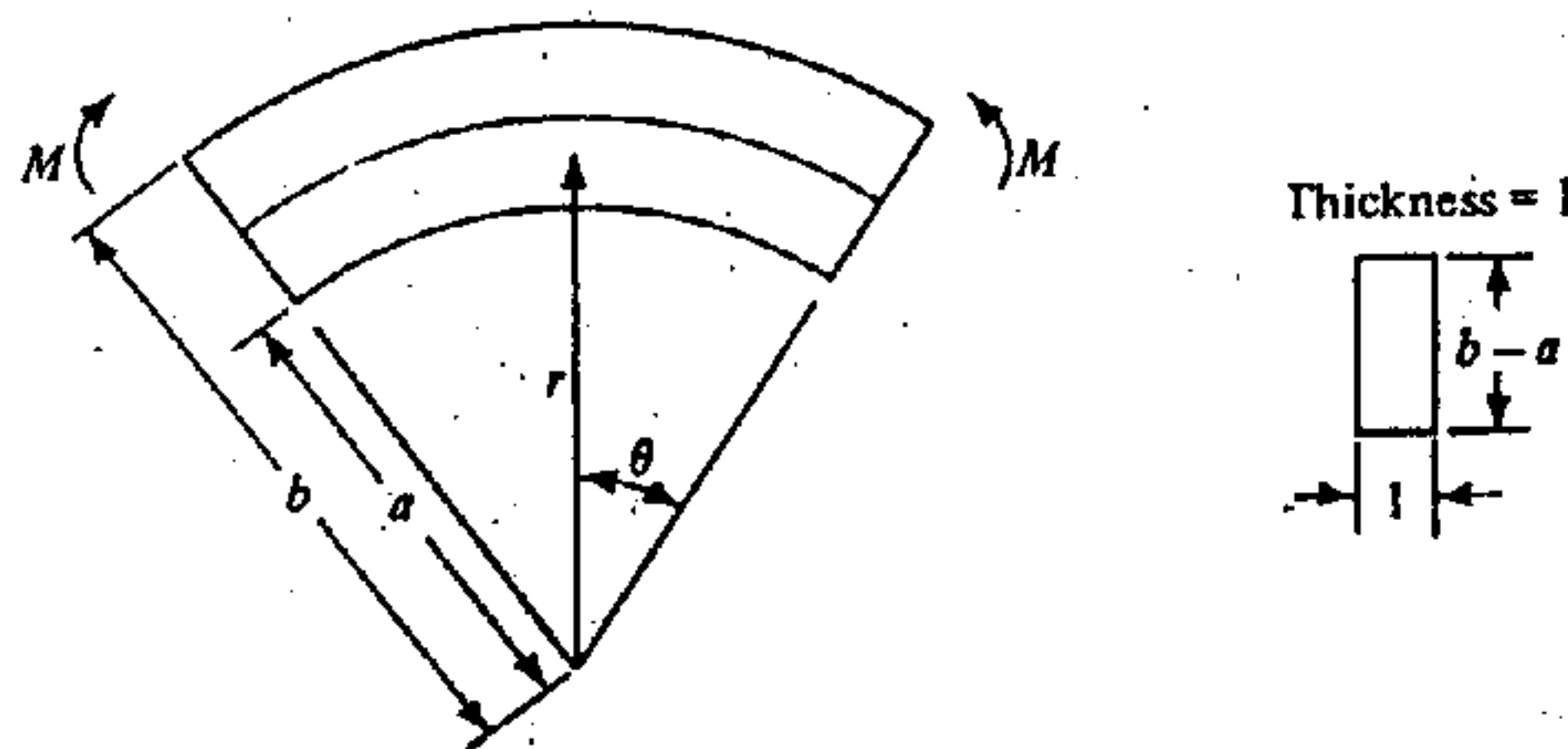
where EI , ν , h and P are constants. Determine if the strains are compatible. (15%)
- The simply-supported beam shown in the figure is loaded by its own weight (with the specific weight ρg in the y -direction).

(1) Show that $\phi = Ax^2y^3 + By^5 + Cy^3 + Dx^2y$ is a plane stress function. (8%)

(2) Determine the coefficients A , B , C and D , and the stress components. (17%)



- Consider the circular bar shown limited by two concentric circles with radii a and b . The thickness is small as compared with its height $b-a$. Two equal and opposite couples M are applied at the ends of the bar. Find the stress components of the circular bar by using the Airy's stress function method. (25%)



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