

國立臺灣科技大學
八十九學年度博士班招生考試試題

系所組別：工業管理系丁組
科 目：機率與統計

Show intermediate steps and formulas for partial credit. You must explain how you compute your results or answers for full credit.

1. (15 points)

Let X be an arbitrary continuous random variable. Show $E(X) = \int_0^{\infty} P\{X > u\} du = \int_{-\infty}^0 P\{X < u\} du$.

2. (15 points)

Let (X, Y) be a continuous random vector with joint density function $f(x, y) = cxy$ for $0 < x < y, 0 < y < 1$. Find c and $P\{X + Y < 1\}$.

3. (20 points)

Suppose that n independent trials are performed. Each trial results in a "type I failure" with probability p_1 or in a "type II failure" with probability p_2 . Let X and Y be the numbers of type I and II failures in n independent trials.

- (a) (10 points) Find the conditional expectation of Y given $X=x$.
(b) (10 points) Find the covariance of X and Y .

4. (15 points)

Consider a large manufactured lot of items of which a fraction q are defective where $0 < q < 1$. Using the conditional expectation argument, find the expected number of items that must be sampled to obtain a total of m non-defective items.

5. (15 points)

(a) (5 points) Fill in the blanks in the following ANOVA table.

Source	SS	df	MS	F
Regression	_____	1	28.9	_____
Error	20.3	_____	_____	_____
Total	_____	4	_____	_____

- (b) (5 points) Compute R^2 .
(c) (5 points) If $\hat{\sigma}_x = 10$, estimate the regression parameter $\hat{\beta}_1$.

6. (20 points)

The generating function of a probability mass function $p_k = P\{X = k\}$ for $k=0,1,\dots$ is defined by

$$g_X(s) = E[s^X] = \sum_{k=0}^{\infty} p_k s^k \text{ for } |s| < 1.$$

- (a) (10 points) Find the generating function for a Poisson random variable X with parameter α .
(b) (10 points) Let X and Y be independent Poisson random variables with parameters α and β , respectively. Find the generating function for a Poisson random variable $Z=X+Y$.

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