

國立臺灣科技大學

九十一學年度博士班招生考試試題

系所組別：機械工程系甲一組

科目：彈性力學

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1. (a) Show that the following strain components of a plane-stress problem is a possible state of small strain,

$$\epsilon_{xx} = xy, \quad \epsilon_{yy} = -\nu xy, \quad \epsilon_{xy} = \frac{(1+\nu)}{2}(b^2 - y^2). \quad (b \text{ and } \nu \text{ are constants}) \quad (7\%)$$

- (b) Find the displacements of the plane-stress problem. (18%)

2. A square plate is loaded as shown in Figure 1. Determine:

- (a) the principal stresses and the corresponding principal directions, (13%)

- (b) the normal and the shearing stresses on section A-A. (12%)

3. A beam having a narrow rectangular cross-section of unit width rotates about the origin at a constant angular speed Ω , and all the boundaries are traction-free, as shown in Figure 2.

- (a) Show that the stress function ϕ must satisfy

$$\nabla^4 \phi = 2\rho(1-\nu)\Omega^2. \quad (7\%)$$

- (b) If the stress function $\phi = a_1 x^4 + a_2 x^2 y^2 + a_3 y^4 + a_4 x^2 + a_5 y^2$

is the solution to this problem, find the stress field of the problem. (18%)

4. Navier's equation in vector form is $G[\nabla^2 \vec{u} + \frac{1+\nu}{1-\nu} \nabla(\nabla \cdot \vec{u})] + \vec{F} = \vec{0}$.

- (a) Show that for an axisymmetric problem, Navier's equation reduces to

$$\frac{2G}{1-\nu} \left\{ \frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} (ru_r) \right] \right\} + F_r = 0. \quad (10\%)$$

- (b) Use (a) to find the stresses and the displacements in an infinite elastic medium which contains a circular cylindrical hole of radius a and is subjected to an internal constant pressure p on the boundary. (15%)

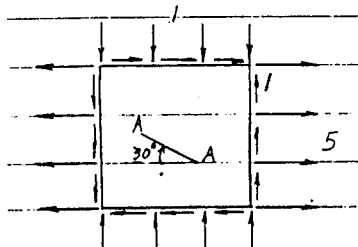


Figure 1.

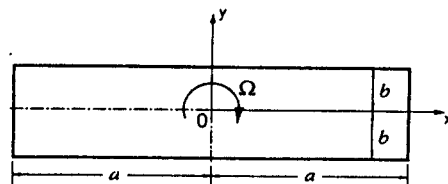


Figure 2.

