國立臺灣科技大學

九十一學年度博士班招生考試試題

系所組別：工業管理系丁組

科　目：機率與統計

總分 100 分

Show intermediate steps and formulas for partial credit. You must explain how you compute your results or answers for full credit.

1. (20 points)

Let X and Y be continuous random variables having joint density $f(x,y)=n(n-1)(y-x)^{n-2}$ for $0\le x\le y\le 1$.

(a) (10 points) Find the conditional density and conditional expectation of Y given $X=x$.

(b) (10 points) Find the expected value of Y.

2. (20 points)

Suppose that U and V are independent and follow the geometric distribution

$f(k)=p(1-p)^k$ for $k=0,1,2,\ldots$

Define the random variable $Z=U+V$.

(a) (10 points) Find the joint probability mass function of U and Z.

(b) (10 points) Find the conditional probability mass function for U given that $Z=n$.

3. (10 points)

Suppose that a random variable X is distributed according to a Poisson distributed with parameters λ. The parameter λ is exponentially distributed with density $f(x)=\theta e^{-\theta x}$ for $x\ge 0$. Find the probability mass function for X.

4. (30 points)

(a) Use the method of least squares of fit a straight line to the $n=5$ data points given in Table

x	y
−2	0
−1	0
0	1
1	1
2	3

(b) Find the variances of the estimators $\hat{\beta}_0$ and $\hat{\beta}_1$

(c) Estimate σ^2 from the data

(d) Calculate a 95% confidence interval for the parameter β_2

5. (20 points)

If $X_1, X_2, \ldots, X_n$ constitute a random sample of size n from a Bernoulli population, show that

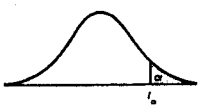
$$\hat{\theta}=\frac{X_1+X_2+\cdots+X_n}{n}$$

is a sufficient estimator of the parameter θ.

國立臺灣科技大學
九十一學年度博士班招生考試試題

系所組別：工業管理系丁組
科　目：機率與統計

Table 5
Percentage points of the t distributions



$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	d.f.
3.078	6.314	12.706	31.821	63.657	1
1.886	2.920	4.303	6.965	9.925	2
1.638	2.353	3.182	4.541	5.841	3
1.533	2.132	2.776	3.747	4.604	4
1.476	2.015	2.571	3.365	4.032	5
1.440	1.943	2.447	3.143	3.707	6
1.415	1.895	2.365	2.998	3.499	7
1.397	1.860	2.306	2.896	3.355	8
1.383	1.833	2.262	2.821	3.250	9
1.372	1.812	2.228	2.764	3.169	10
1.363	1.796	2.201	2.718	3.106	11
1.356	1.782	2.179	2.681	3.055	12
1.350	1.771	2.160	2.650	3.012	13
1.345	1.761	2.145	2.624	2.977	14
1.341	1.753	2.131	2.602	2.947	15
1.337	1.746	2.120	2.583	2.921	16
1.333	1.740	2.110	2.567	2.898	17
1.330	1.734	2.101	2.552	2.878	18
1.328	1.729	2.093	2.539	2.861	19
1.325	1.725	2.086	2.528	2.845	20
1.323	1.721	2.080	2.518	2.831	21
1.321	1.717	2.074	2.508	2.819	22
1.319	1.714	2.069	2.500	2.807	23
1.318	1.711	2.064	2.492	2.797	24
1.316	1.708	2.060	2.485	2.787	25
1.315	1.706	2.056	2.479	2.779	26
1.314	1.703	2.052	2.473	2.771	27
1.313	1.701	2.048	2.467	2.763	28
1.311	1.699	2.045	2.462	2.756	29
1.282	1.645	1.960	2.326	2.576	inf.

From "Table of Percentage Points of the t-Distribution." Computed by Maxine Merrington, *Biometrika*, Vol. 32 (1941), p. 300. Reproduced by permission of Professor E. S. Pearson.