

國立臺灣科技大學

九十二學年度博士班招生考試試題

系所組別：電機工程系博士班乙一組

科目：信號與系統

總分100分

1. Determine whether the following systems are causal or noncausal, are linear or nonlinear, are time-invariant or time-varying. In the following parts,  $x(t)$  (or  $x[n]$ ) is an arbitrary input and  $y(t)$  (or  $y[n]$ ) is the zero-state response to  $x(t)$  (or  $x[n]$ ). (20%)

(a)  $y(t) = \int_{-\infty}^t t \cdot x(\lambda) d\lambda$

(b)  $y[n] = u[n] \cdot x[n]$ , where  $u[n]$  denote the unit-step function.

2. Given two real numbers  $a$  and  $b$ , the ratio  $a/b$  can be computed by recursively solving the difference equation

$$y[n+1] + (b-1)y[n] = a, \quad n \geq 0 \quad (20\%)$$

- (a) Derive an analytical expression for the solution  $y[n]$  assuming an arbitrary initial condition  $y[0]$ .
- (b) Under what conditions does  $y[n]$  converge to  $a/b$  as  $n \rightarrow \infty$ ?

3. A linear time-invariant continuous-time system has impulse response

$$h(t) = e^{-t} + \sin(t), \quad t \geq 0 \quad (20\%)$$

- (a) Compute the step response  $g(t)$  for all  $t \geq 0$ .
- (b) Compute the output response  $y(t)$  for all  $t \geq 0$  when the input is  $u(t) - u(t-2)$  with no initial energy in the system at time  $t = 0$ .

4. A linear time-invariant continuous-time system has the frequency response function

$$H(\omega) = \frac{1}{j\omega + 1} \quad (20\%)$$

- (a) Plot the amplitude and phase functions for  $H(\omega)$ .
- (b) Compute the output response  $y(t)$  when the input  $x(t) = \cos(t+\pi/4)$ .

5. The autocorrelation of a discrete-time signal  $x[n]$  is defined by

$$R_x[n] = \sum_{i=-\infty}^{\infty} x[i]x[n+i] \quad (20\%)$$

Let  $P_x(\Omega)$  denote the DTFT of  $R_x[n]$  and  $X(\Omega)$  denote the DTFT of  $x[n]$ .

- (a) Derive an expression for  $P_x(\Omega)$  in terms of  $X(\Omega)$ .
- (b) Express  $P_x(0)$  in terms of  $x[n]$ .

