國立臺灣科技大學
九十三學年度博士班考試試題

系所組別：資訊管理系甲組
科　　目：資料結構

【本試卷共六大題，滿分 100 分。】

1. (a) There are three disjoint sets which are represented by trees as shown in Fig.1. Show the parent array which represents the sets. (5%)
   (b) Explain how to apply the two operations "find" and "union" of disjoint set structure in Kruskal's algorithm, which finds the minimum cost spanning tree in a graph. (5%)
   (c) Prove that using weighting rule to merge disjoint set structures can control the height of the tree no greater than $\lfloor \log_2 n \rfloor + 1$. (5%)

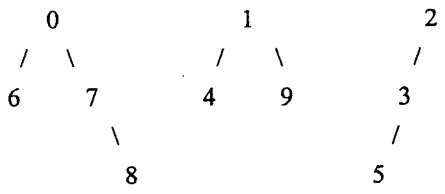

Fig. 1

2. (a) Suppose data are inserted into an initially empty Deap in the sequence of 4, 2, 6, 5, 1, 8, 10, 7, 3, 9. Draw the corresponding Deaps. (5%)
   (b) Show the final AVL tree and red-black tree after successively inserting the keys 2, 7, 8, 1, 3, 6, 4, 5, 9 into an initially empty AVL and red-black trees. (10%)

3. (a) We have a set of n jobs to be executed, each of which takes unit time. At any instant t, we can execute exactly one job and earns a profit g_i if and only if it is executed no later than time d_i for $1 <= i <= n$. Write the algorithm which schedules the jobs such that the maximum profit can be obtained. In addition, describe the data structures you use. (15%)
   (b) Analyze its time complexity. (5%)

4. (a) Describe the Dijkstra's algorithm for generating single-source shortest paths. (10%)
   (b) Use Dijkstra's algorithm to find the shortest paths from node v0 to each of v1, v2, v3, v4 and v5 in Figure 2. (10%)

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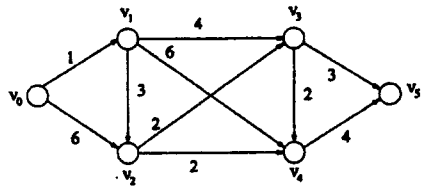

Figure 2

5. Construct an optimal Huffman code for the set of letters in the table. (10%)

Letter	Frequency
a	2
b	3
c	5
d	8
e	13
f	21

6. Compute the Big Oh(i.e., the tight bound) of the following recurrences, and order them according to their growth rates (from high to low). (20%)

a. $T_1(n) = T_1(n-1) + 1/n$, $T_1(1) = 1$
b. $T_2(n) = T_2(n-1) + n$, $T_2(1) = 1$
c. $T_3(n) = 2T_3(n/2) + 1$, $T_3(1) = 1$
d. $T_4(n)=8T_4(n/2) + 6n$, $T_4(1)=6$