

國立臺灣科技大學

九十四學年度博士班招生考試試題

系所組別：機械工程系甲二組

科目：最佳設計

總分 100 分

1. For the function $f(x_1, x_2) = 5x_1 - \frac{1}{16}x_1^2x_2 + \frac{1}{4x_1}x_2^2$.
- Find stationary points. (5%)
 - Calculate the Hessian matrices at the stationary points. (5%)
 - Determine if they are local minimum, local maximum, or inflection points for the function. (5%)
 - Calculate the gradient vector at the point (1, 2). (5%)
 - Check if the direction $d = (1, -2)$ at the point (1, 2) is a descent direction for the function. (10%)
2. Minimize $f(x_1, x_2) = (x_1 - 10)^2 + (x_2 - 5)^2$
- subject to $x_1 + x_2 \leq 12, x_1 \leq 8, x_1 - x_2 \leq 4$ (10%)
- Check if the minimum point satisfies Kuhn-Tucker necessary conditions. (10%)
3. Find the optimal value of x_1 and x_2 which minimize the object function $U = x_1^2 + x_2^2$ and subject to the equality constraint $(x_1 - 8)^2 + (x_2 - 6)^2 - 25 = 0$. (25%)
4. A total of 300 lineal feet of tubes must be installed in a heat exchanger in order to provide the necessary heat-transfer surface area. The total dollar cost of the installation includes:
- The cost of the tubes, \$700
 - The cost of the shell = $25D^{2.5}L$
 - The cost of the floor space occupied by the heat exchanger = $20 DL$
- where D and L are respectively the diameter and the length of the heat exchanger. The spacing of the tubes is such that 20 tubes should be fitted in a cross-sectional area of 1 ft^2 . Determine the optimal value of the design variables D and L such that the purchase cost is minimized. (25%)

