國立臺灣科技大學
九十四學年度博士班招生考試試題
系所組別：工業管理系丁組
科　　目：機率與統計

總分100分

1. (10 points) Three shops *A, B,* and *C* produce respectively *50%, 30%,* and *20%* of the total number of items of a factory. The percentages of defective output of these shops are *3%, 4%,* and *5%*.
   (a) (5 points) Suppose an item is selected at random and is found to be defective. Find the probability that the item was produced by the shop *A*.
   (b) (5 points) By changing the process, the percentage of defective output of the shop *C* reduces to *p %* and the probability of producing a defective item of the factory reduces to *3.3%*. What is the value of *p*?

2. (20 points) If *X* and *Y* are independent random variables where *X* is uniformly distributed on [0, 1] and *Y* is uniformly distributed on [1, 2]. That is, $f_X(x)=\begin{cases}1, & 0\le x\le 1\\ 0, & otherwise\end{cases}$ and $f_Y(y)=\begin{cases}1, & 1\le y\le 2\\ 0, & otherwise\end{cases}$.
   (a) (10 points) If $Z=X+Y$, what is the probability density function of *Z*?
   (b) (10 points) What is $E[Z]$?

3. (20 points) To be a PhD candidate in the IM department of NTUST, a student has to pass an entrance exam and a qualify exam. Suppose that there are *N* students registered the entrance exam and each can pass the exam with probability *p* independently. Only those who past the entrance exam can take the qualify exam and each can pass the qualify exam with probability *q* independently. Let *M* be the number of students past the entrance exam and *X* be the number of PhD candidates among the *N* students.
   (a) (10 points) What is the joint pmf of *X* and *M*?
   (b) (10 points) What is the pmf of *X* and what is the expected value of *X*?

4. (25 points) Let $\overline{X}=\frac{\sum_{i=1}^{n}X_i}{n}$ and $S^2=\frac{\sum_{i=1}^{n}(X_i-\overline{X})^2}{n-1}$ be two statistics computed from a Normal random sample with both parameters μ and σ^2 unknown.
   (a) (5 points) List your statistic and decision rules to test the hypothesis $\mathrm{H}_0:\mu=\mu_0$ v.s. $\mathrm{H}_1:\mu\ne\mu_0$ at $\alpha=0.05$.
   (b) (8 points) Use the Markov's inequality to find an appropriate size *n* that satisfies $Pr(\overline{X}-\mu\ge 0.1\sigma)\le 0.1$.
   (c) (5 points) List your statistic and decision rules to test the hypothesis $\mathrm{H}_0:\sigma^2={\sigma_0}^2$ v.s. $\mathrm{H}_1:\sigma^2\ne{\sigma_0}^2$ at $\alpha=0.05$.
   (d) (7 points) Use the Markov's inequality to find an appropriate size *n* that satisfies $Pr(\ |S^2-\sigma^2|\ \ge\ 0.1\sigma^2)\le 0.1$.

5. (25 points) Consider the following statistical model:

$$\mathrm{Y}_{ij}=\mu+\tau_i+\beta_j+\varepsilon_{ij},\quad i=1,2,\ldots,a;\ \ j=1,2,\ldots,b$$

   (a) (5 points) $\mathrm{Var}(\mathrm{Y}_{ij})$=?
   (b) (5 points) $\mathrm{Cov}(\mathrm{Y}_{ij},\ \mathrm{Y}_{ik})$=? (for $j\ne k$)
   (c) (7 points) $\mathrm{E}[\sum_{i=1}^{a}(\mathrm{Y}_{ij}-\overline{\mathrm{Y}}_{\bullet j})^2]$=? (where $\overline{\mathrm{Y}}_{\bullet j}=\sum_{i=1}^{a}\mathrm{Y}_{ij}/a$)
   (d) (8 points) $\mathrm{E}[\sum_{j=1}^{b}(\mathrm{Y}_{ij}-\overline{\mathrm{Y}}_{i\bullet})^2]$=? (where $\overline{\mathrm{Y}}_{i\bullet}=\sum_{j=1}^{b}\mathrm{Y}_{ij}/b$)

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