國立台灣科技大學九十五學年度博士班招生試題

系所組別： 工業管理系博士班甲組

科　　目： 作業研究

Total 100 points

This exam is composed of 4 questions and totals 100 points. The point value for each question is listed next to the question. Budget your time accordingly. Read the entire exam before you begin. Start with the questions that you can solve quickly and correctly.

1. (25 points) The Chair Company has three plants producing chairs that are to be shipped to four distribution centers. Plants 1, 2, and 3 produce 11, 17, and 12 shipments per month, respectively. Each distribution center needs to receive 10 shipments per month. The distance (in miles) from each plant to the respective distributing centers is given in the following table:

	Distribution Center			
Plant	1	2	3	4
1	820	1300	500	700
2	1000	1400	700	900
3	650	1200	900	800

The freight cost for each shipment is $120 plus $0.6 per mile. How much should be shipped from each plant to each of the distribution centers to minimize the total shipping cost?

(a) Formulate this problem as a transportation problem by constructing the appropriate parameter table.

(b) Draw the network representation of this problem.

(c) Use the northwest corner rule to construct an initial basic feasible solution.

2. (25 points) A city has experienced a higher than average crime rate in the past few years. There are three primary crime districts in the city. The police chief must allocate five additional patrol teams to these districts. The number of crimes that could be prevented per month by assigning additional patrol teams to the districts is given in the following table:

	Number of additional patrol teams					
Districts	0	1	2	3	4	5
A	0	12	22	24	12	15
B	0	16	21	23	16	19
C	0	13	21	25	13	20

Use dynamic programming to determine the optimal assignment of patrol teams.

3. (25 points) Consider the Markov chain $\{X_n\}$ with states {1, 2, 3, 4, 5, 6, 7} and transition matrix

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0.3 & 0 & 0 & 0.6 \\ \beta & 0 & 0 & 0 & \alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0 & 0.6 \\ 0 & 0 & 0 & 0.4 & 0 & 0.6 & 0 \end{bmatrix}$$

Suppose $X_0 = 5$, find α and β such that the long-term probability of being in state 5 is 1/2.

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4. (25 points) A CTMC (Continuous Time Markov Chain) $\{Y_t\}$ can be constructed as follows: Take an irreducible discrete time Markov chain $\{X_n\}$ with transition matrix P (P may have infinite dimension) and a set of rates $\{r(i)\}$.The CTMC stays in state i for a time that is exponentially distributed with $r(i)$. The next state is then chosen according to P. Show that $p_k = \lim_{t\to\infty}\{Y_t = k\}$ is related to $\pi_k = \lim_{n\to\infty}\{X_n = k\}$ by $p_k = \dfrac{\pi_k / r(k)}{\sum_j \pi_j / r(j)}$.