

國立台灣科技大學九十五學年度博士班招生試題

系所組別：工業管理系博士班丁組

科目：機率與統計

Total 100 points

Show intermediate steps and formulas for partial credit. You must explain how you compute your results or answers for full credit.

1. (30 points)

Given any p and N , let X have a binomial distribution with parameter p and N . What is the resulting distribution of X under each of the following conditions?

(a) (15 points) Suppose that p is uniformly distributed on the interval $[0,1]$ for any fixed N .(b) (15 points) For each fixed p , suppose that N follows the geometric distribution with

$$p_N(n) = (1-\beta)\beta^{n-1}, \text{ for } n=1,2,\dots$$

2. (20 points)

Suppose a pair of dice are tossed and X_i is a random variable whose value is the result on die i , $i=1,2$. Let S be the random variable whose value is the sum on the dice ($S = X_1 + X_2$).

(a) (5 points) Find $P\{S=6\}$.(b) (5 points) Find $P\{X_1=j|S=6\}$ for $j=1,2,3,4,5$.(c) (5 points) Find $P\{S=6|X_1=j\}$ for $j=1,2,3,4,5$.(d) (5 points) Find $E(X_1|S=6)$.

3. (25 points)

Suppose that $X_i \sim N(\mu, \sigma^2)$, where σ^2 is known, and we wish to test $\begin{cases} H_0: \mu = \mu_0 \\ H_1: \mu \neq \mu_0 \end{cases}$.

Please find the generalized likelihood ratio (GLR) test and determine the critical region for the GLR statistic.

4. (25 points)

If A is a set, then the indicator function of A , denoted by I_A , is defined as $I_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$.

Let $A = (0, \theta)$, then $f(x; \theta) = \frac{1}{\theta} I_{(0, \theta)}(x)$. Please find the statistic is sufficient for θ .