

國立台灣科技大學九十六學年度博士班招生試題

系所組別：工業管理系博士班甲組

科目：作業研究

總分 100 分

1. Consider the following Linear Programming problem:

$$\text{Minimize } z = x_1 + 2x_2 + x_3$$

Subject to

$$x_1 + x_2 - x_3 \leq 2$$

$$x_1 - x_2 + x_3 \geq 1$$

$$x_1, x_2, x_3 \geq 0$$

- (1) Use the big-M method to find the optimal value and optimal solution for this problem. (10 %)
 (2) Is the region bounded for these constraints? Explain your answer clearly. (10 %)
 (3) Give the dual problem for this question. (5 %)

2. The following problem is formed by some company's manager:

$$\text{Maximize } z = cx_1 + x_2$$

Subject to

$$x_1 - 2x_2 \leq 0$$

$$2x_1 - x_2 \geq 0$$

$$x_1 + x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

- (1) Use the graph method to find the optimal solution if $c = 3$. (7 %)
 (2) If the manager was told that c should choose from the interval $[0, 2]$, then discuss the optimal value and the corresponding optimal solution(s) at different values of c . (18 %)
3. (25 %) Three balls are divided between two containers. During each period, a ball is randomly chosen and switched to the other container.
- (1) In steady state, find the fraction of time that a container will contain 0, 1, 2 or 3 balls.
 (2) If container 1 contains no balls, on the average how many periods will go by before it again contains no balls?
4. (25 %) Suppose product i ($i = 1, 2$) cost $\$c_i$ per unit. If x_i ($i = 1, 2$) units of products 1 and 2 are purchased, then a utility $x_1^a x_2^{1-a}$ ($0 < a < 1$) is received.
- (1) If $\$d$ are available to purchase products 1 and 2, how many of each type should be purchased?
 (2) Show that an increase in the cost of product i decreases the number of units of product i that should be purchased.
 (3) Show that an increase in the cost of product i does not change the number of units of the other product that should be purchased.