國立台灣科技大學九十六學年度博士班招生試題

系所組別： 工業管理系博士班丁組

科　　目： 機率與統計

Show intermediate steps and formulas for partial credit. You must explain how you compute your results or answers for full credit. 滿分為100分

1. (25 points) (計算至小數點第 4 位)

   Let X be the number of flaws on the surface of a randomly selected boiler of a certain type. Suppose X has a Poisson distribution with mean λ. However, the mean λ is unknown. Past experience indicates that the prior probabilities of λ are $P\{\lambda = 4\} = 0.3$, $P\{\lambda = 5\} = 0.5$ and $P\{\lambda = 6\} = 0.2$.

   (a) (5 points) Given that the true mean of X is 5, what is the probability that a randomly selected boiler has at most two flaws?

   (b) (10 points) Under the prior probabilities, what is the probability that a randomly selected boiler has exactly two flaws?

   (c) (10 points) Suppose that a boiler was randomly selected and found that it had exactly two flaws. Using this information, revise the prior probabilities of λ.

2. (25 points) (計算至小數點第 4 位)

   A company distributes boxes of a mixture of materials A , B and C. Suppose that the weight of each box is 1 kg, but the individual weights of the materials A, B and C vary from box to box. For a randomly selected box, let X and Y represent the weights of the material A and B, respectively and suppose that joint density function of X and Y is given as follows.

$$f(x,y) = \begin{cases} 24xy, & 0 \le x \le 1,\ 0 \le y \le 1,\ x + y \le 1 \\ 0, & elsewhere. \end{cases}$$

   (a) (5 points) Find the probability that the weight of material C is more than 0.5 kg in a given box.

   (b) (10 points) Find the probability that the weight of material B is less than 0.125 kg if the weight of material A is 0.75 kg in a box.

   (c) (10 points) Find the probability density function of the random variable $Z=X+Y$.

3. (30 points)

   (a) (15 points) Please describe the Neyman-Pearson Lemma for the most powerful test and prove it.

   (b) (15 points) Find the form of the most powerful test of $H_0 : p = p_0$ against $H_a : p = p_1 > p_0$ based on the statistic S~Binomial distribution (n, p).

4. (20 points)

   Find the MLE (maximum likelihood estimation) for θ based on a sample of size n from a distribution with pdf (probability density function)

$$f(x;\theta) = \begin{Bmatrix} 2\theta^2 x^{-3} & \theta \le x \\ 0 & x < \theta; 0 < \theta \end{Bmatrix}$$