

國立台灣科技大學九十五學年度碩博士在職專班招生試題

系所組別：管理研究所EDBA博士在職專班甲組、乙組、丙組、丁組

科目：統計學

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統計學

共七題，總計 100 分。依序作答。

1. The probability of getting a head when flipping a biased coin is 0.6. Suppose that the results of flips are statistically independent. Give the expressions, instead of the numerical values, when answering the following questions.
 - (a) (5%) What is the probability of getting 3 heads when flipping the biased coin 10 times.
 - (b) (5%) What is the probability of getting 3 heads in less than 10 flips?
2. A bag contains 4 balls with different colors. Draw 5 balls one by one with replacement. Let N denote the number of colors observed in the 5 balls drawn.
 - (a) (5%) Give the expression for $E(N)$.
 - (b) (5%) Give the expression for the average number of draws required to observe all 4 colors.
3. (10%) Give one example of the *sample statistic*. What is the purpose of calculate a *sample statistic*? Why is *sample statistic* a random variable? Why do we need to learn its *sampling distribution*?
4. (10%) A coin is flipped to test if it is biased. Suppose that the 5th head appears in the 30th flip. Our goal is to test the hypothesis that the probability of getting a head of this coin is $P(\text{head}) = 0.2$. To answer this question, what probability would you calculate? Your answer should include (a) random variable definition, (b) event definition (using the defined random variable), (c) probability distribution of the defined random variable (including parameters of the probability distribution), (d) probability expression (not the numerical value), and (e) how to draw the conclusion based on the calculated probability.
5. (10%) Suppose that random sample $\{X_1, \dots, X_{25}\}$ is drawn from $N(\mu, \sigma^2 = 2^2)$ with μ unknown. The goal is to test $H_0 : \mu = 32$ against $H_1 : \mu < 32$. If the *rejection region* is

$$\left\{ (x_1, \dots, x_{25}) \mid \bar{x} = \sum_{i=1}^{25} x_i / 25 \leq 31.2 \right\}.$$

Find the *power* of this test at $\mu = 30.8$.(You may need $P(|Z| < 1) = 68\%$, $P(|Z| < 2) = 95\%$, where Z is the standard normal random variable.)

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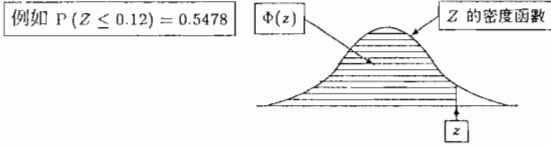
6. (25%) A random sample of $n = 40$ observations from a population produced a mean $\bar{X} = 83.8$ and a standard deviation equal to 2.9. Suppose that you wish to show that the population mean μ is less than 84.
- Give the alternative hypothesis for the test.
 - Give the null hypothesis for the test.
 - If you wish your probability of (erroneously) deciding that $\mu < 84$, when in fact $\mu = 84$, to equal 0.05, what is the value of α for the test?
 - Before you conduct the test, glance at the data and use your intuition to decide whether the sample mean $\bar{X} = 83.8$ implies that $\mu < 84$. Now test the null hypothesis. Do the data provide sufficient evidence to indicate that $\mu < 84$? Test using $\alpha = 0.05$.
 - Find the critical value of \bar{X} necessary for rejection of H_0 .
 - Calculate β when $\mu = 82.8$. Repeat the calculation for $\mu = 82.4, 82.6$ and 83.4 .
 - Use the values of β calculated in part (f) to graph the power curve for the test.
7. (25%) Ryan's Trucking Company determined that on an annual basis, the distance traveled per truck is normally distributed with a mean of 50.0 thousand miles and a standard deviation of 12.0 thousand miles. If a sample of 16 trucks is selected,
- What is the probability that the average distance traveled is below 45.0 thousand miles in the year?
 - What is the probability that the average distance traveled is between 44.0 and 48.0 thousand miles in the year?
 - What assumption must be made in order to solve (a) and (b)?
 - If a random sample of 64 trucks is selected, there is a 95% chance that the sample mean will be below how many miles?
 - What assumption must be made in order to solve (d)?
 - What will your answers be to (a)-(d) if the standard deviation is 10.0 thousand miles?



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標準常態分布累積機率函數 $\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = P(Z \leq z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990



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