

國立台灣科技大學九十七學年度碩博士在職專班招生試題

系所組別：電機工程系博士在職專班已組

科目：工程數學

*Total 100 points*

1. (10%) Solve the following differential equation for
- $y(x)$

$$x^2 y'' - 3xy' + 3y = 3(\ln x)^2 + \ln x - 1$$

2. (10%) Solve the following differential equation for
- $y(x)$
- upon the initial condition

$$y''' + 4y'' + 5y' + 2y = 0, \quad y(0)=1, y'(0)=y''(0)=0$$

3. (10%) Solve the following differential equation for
- $y(t)$
- upon the initial conditions

$$y'' + 3y' + 2y = 3\delta(t-2), \quad y(0) = y'(0) = 0$$

4. (10%) Determine the expression that gives all values of
- $(\sqrt{3} + i)^{3i}$
- .

5. Use the residue theorem to evaluate the integrals over the paths:

(a) (5%)  $\oint_{\Gamma} e^{\frac{1}{z}} dz$ ,  $\Gamma$  is the circle of radius 1 about  $-2i$

(b) (5%)  $\oint_{\Gamma} e^{\frac{1}{z}} dz$ ,  $\Gamma$  is the circle of radius 1 about 0

6. (10%) Let
- $A$
- and
- $B$
- be
- $3 \times 3$
- matrices with
- $\det(A) = 4$
- and
- $\det(B) = 5$
- . Find the value of:

(a)  $\det(AB)$

(b)  $\det(3A)$

(c)  $\det(A^{-1}B)$

7. (10%) Given

$$x_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

- (a) Show that
- $x_1, x_2, x_3$
- are linearly dependent.

- (b) What is the dimension of
- $\text{Span}(x_1, x_2, x_3)$
- ?



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科 目：工程數學

8. (10%) Consider  $A = \begin{pmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{pmatrix}$

(a) Find the eigenvalues and the corresponding eigenvectors.

(b) Find  $A^n$  where  $n$  is a positive integer.

9. (10%) Consider a well-shuffled deck of cards consisting of 52 distinct cards, of which four are aces and four are kings.

(a) Find the probability of obtaining an ace in the first draw.

(b) Suppose we draw 7 cards from the deck. What is the probability that the 7 cards include 3 aces?

(c) Suppose that the entire deck of cards is distributed equally among four players. What is the probability that each player gets an ace? (10%)

10. (10%) The random variables  $X$  and  $Y$  have the joint pdf

$$f_{X,Y}(x,y) = 2e^{-(x+y)}, \quad 0 \leq y \leq x < \infty.$$

(a) Find the marginal pdf's  $f_X(x)$  and  $f_Y(y)$ .

(b) Are  $X$  and  $Y$  independent? Explain your answer.

